

# Device Correlation: Modeling using Uncorrelated Parameters, Characterization Using Ratios and Differences

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## ABSTRACT

Partial correlations between parameters of different types of devices, such as effective channel length for PMOS and NMOS devices, are often modeled and simulated statistically via correlation coefficients. However this is cumbersome and inefficient from a modeling perspective, from a characterization perspective, and from a simulation perspective. We have found that, with physical understanding, it is possible to formulate models with a combination of parameters that are common to, and completely correlated between, different types of devices and parameters that are unique to, and completely independent between, different types of devices. This gives a modeling basis of independent statistical parameters, which is ideal for simple statistical simulation, yet completely encompasses statistical correlations between different device types. The key then is how these underlying parameters, which are not directly observable, can be characterized from measurements. We show that by identifying suitable ratios or differences of electrical measurements between device types, that “hidden” physical parameters, not observable from direct measurements of a single type of device or electrical performance, can be easily characterized statistically. We show that the technique gives error free values for the variances of correlated parameters, and allows oxide thickness variation to be characterized from simple DC measurements. As far as we aware, this is the first non-tunneling approach that allows oxide thickness variations to be determined from DC only measurements.

**Keywords:** MOSFET modeling; SPICE modeling; statistical Modeling.

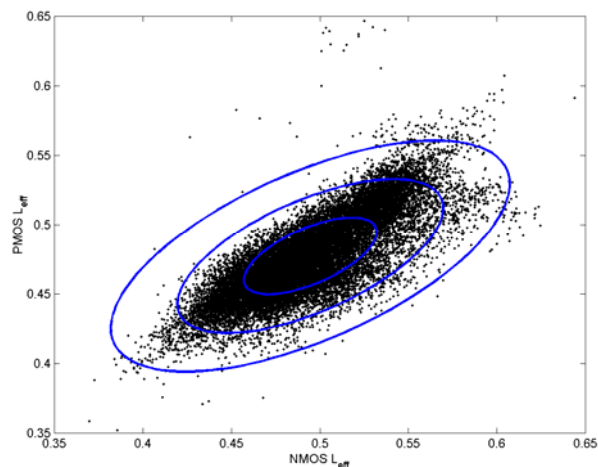
## 1. INTRODUCTION

The electrical properties of devices in semiconductor manufacturing processes are predominantly controlled by a small number of physical process parameters. For MOSFETs these include oxide thickness  $T_{ox}$ , flatband voltage  $V_{fb}$ , channel length offset  $\Delta_L = L - L_{eff}$  (where  $L$  is design channel length and  $L_{eff}$  is effective electrical channel length), channel width offset  $\Delta_W = W - W_{eff}$ , and effective substrate doping  $N$ . Here we consider only MOSFETs, but the principles are applicable to all device types. We also consider only the statistical variations of these parameters.

Some device level process parameters can be correlated between device types, such as  $L_{eff}$  for PMOS and NMOS devices. Fig. 1 shows PMOS  $L_{eff}$  plotted against NMOS  $L_{eff}$  for a 0.5 $\mu$ m BiCMOS technology. The measurements are from scribe-grid process control (SGPC) data taken over a 6 month

period, and include more the 20,000 samples. Some, but not perfect, correlation is clear.

As will become apparent below, the statistical analyses presented in this paper are targeted at characterizing the global components of statistical variation. The accuracy of the results is predicated on the local (mismatch) component of variation being small relative to the global variation. The local component of variation of a parameter increases as device geometry decreases, and for sub-130nm technologies the local variation can dominate the global variation. Therefore the data used to verify the analyses presented here are for a 0.5 $\mu$ m BiCMOS technology, for which the global component of variation dominates the local component of variation for all device geometries. The analyses are applicable to more advanced technologies, but only if the devices from which the data are derived are large enough so the global component of variation dominates the local component variation.

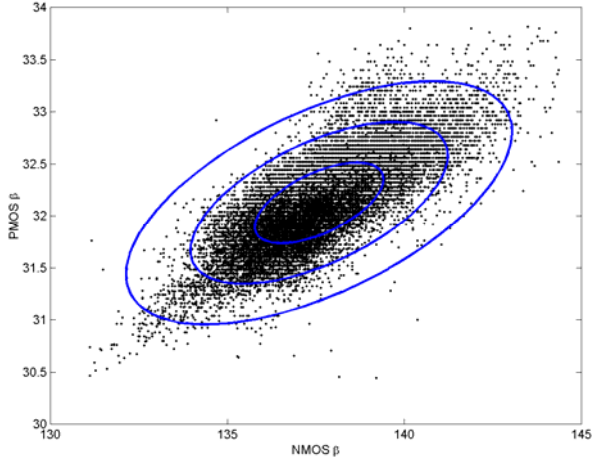


**Fig. 1** Correlation between PMOS  $L_{eff}$  and NMOS  $L_{eff}$ . Points are measured data, ellipses are from fitted model.

Other examples of quantities that can be correlated between PMOS and NMOS devices are the transconductance (or gain) factor  $\beta$  (calculated from the peak  $g_m$  at low  $V_{ds}$ ) of both wide/long and wide/short devices, and the zero bias threshold voltage  $V_{t0}$  of wide/long devices. Fig. 2 shows PMOS  $\beta$  plotted against NMOS  $\beta$  for wide/long devices; again the partial correlation is clear.

Many types of circuits have important measures of electrical performance that depend on correlated, or differential, performance between PMOS and NMOS devices. For example, the hysteresis of a Schmidt trigger depends on the difference between PMOS and NMOS  $V_{t0}$ , propagation delay difference between different polarity signals depends on the differential

performance between pull-up PMOS and pull-down NMOS devices, and the gain of a CMOS op-amp depends on the ratio of  $g_m$  of the drive devices to the output conductance of the load (and drive) devices. Therefore, statistical simulation of many common circuits must properly account for the correlation between different types of devices.



**Fig. 2** Correlation between PMOS  $\beta$  and NMOS  $\beta$ . Points are measured data, ellipses are from fitted model.

Conventionally, correlations are modeled directly. However, there are some drawbacks to this. First, although some sampling techniques, such as Monte Carlo and Latin Hyper-Cube (LHC) sampling [1], can handle parameter correlations, other useful sampling techniques, such as Hadamard sampling [2], cannot.

Second, as the number of parameters and/or devices with correlations increases, then modeling via correlations requires characterization of correlations between each pair of parameters. The number of correlation coefficients increases as the square of the number of parameters, which is undesirable.

Third, empirically measuring correlation coefficients gives no physical insight into the relative magnitude of the contributions of the underlying physical processes that cause the correlated variations. This precludes identification of the best areas to target for improved process capability.

Consequently, it is desirable to formulate statistical device models in terms of uncorrelated, physical process parameters. We have found that, for the devices and technologies we have looked at (which is CMOS, RF BiCMOS, and complex SmartPower processes from the 2.0 $\mu$ m to 130nm generations), it has always been possible to do this. The difficulty is in characterizing the statistics of the underlying process parameters individually. Measurements on single devices implicitly include the effects of the variations of all process parameters, and do not allow characterization of “hidden” process parameters that are correlated with other device types.

In this paper we propose a solution to this difficulty, for statistical characterization. By forming suitable ratios or differences of measured quantities between different types of devices (or different measurements on one type of device for

BJTs, which are not considered here) the statistics of the individual underlying process parameters can be unraveled. Interestingly, this requires no new measurements, only simple manipulations of existing data. And, significantly and interestingly, it gives values for statistical variations of the correlated parameters that are not affected by measurement errors.

## 2. STATISTICAL MODEL FORMULATION

Models for accurate and efficient statistical characterization of semiconductor devices need to be based on physical process parameters  $P$ . SPICE model parameters may need to be written as functions of the  $P$ . Observed variations in each  $P$  come from three main sources: global (interdie) variations  $\delta P_G$  that are assumed perfectly correlated for all devices within a die; local (intradie, or mismatch) variations  $\delta P_L$  that are assumed perfectly uncorrelated between devices; and (if measured directly) measurement error  $\epsilon_P$ . The total variation is then

$$(1) \quad \delta P = \delta P_G + \delta P_L(g) + \epsilon_P$$

where  $g$  are geometric and layout attributes of the device. Standard SGPC data includes statistical variations from all 3 sources in (1).

## 3. STATISTICAL MODELING AND CHARACTERIZATION USING RATIOS

What are the underlying physical reasons why there is partial correlation between wide/long PMOS and NMOS gain factors in Fig. 2? The gain factors, determined from the peak transconductance  $g_m$  at low  $V_{ds}$ , are

$$(2) \quad \beta_{[p,n]} = \mu_{[p,n]} \epsilon_{ox} W_{eff} / T_{ox[p,n]} L_{eff}$$

where  $\epsilon_{ox}$  is the oxide permittivity and  $\mu_p$  and  $\mu_n$  are low field mobilities for the PMOS and NMOS devices. For wide/long devices the variations in  $L_{eff}$  and  $W_{eff}$  have little effect on  $\beta$  and will be ignored. The 3 statistical parameters that affect  $\beta_p$  and  $\beta_n$  are therefore  $T_{ox}$ , which is highly correlated between PMOS and NMOS device, and  $\mu_p$ , and  $\mu_n$ , which are to first order independent for PMOS and NMOS. No introduction of additional physical parameters is required here as the underlying model already includes the device independent and device dependent parameters.

The question then is how to characterize the 3 underlying physical parameters from the 2 measured gain factors? At first it would seem impossible, because it is an undetermined problem. But it turns out that, for the statistical variations, it is possible.

Besides the direct measurement of  $\beta_p$  and  $\beta_n$ , form

$$(3) \quad \beta_r = \beta_p / \beta_n \approx \mu_p / \mu_n$$

where the approximation assumes identically sized PMOS and NMOS and devices with the same  $T_{ox}$ . From (1) through (3)

the variances of the PMOS and NMOS transconductance factors, and their ratio, follow as

$$(4) \quad \sigma_{\beta_p}^2 = \sigma_{T_{oxG}}^2 + \sigma_{T_{oxpL}}^2 + \sigma_{\mu_{pG}}^2 + \sigma_{\mu_{pL}}^2 + \sigma_{\varepsilon_{\beta p}}^2$$

$$(5) \quad \sigma_{\beta_n}^2 = \sigma_{T_{oxG}}^2 + \sigma_{T_{oxnL}}^2 + \sigma_{\mu_{nG}}^2 + \sigma_{\mu_{nL}}^2 + \sigma_{\varepsilon_{\beta n}}^2$$

$$(6) \quad \sigma_{\beta_r}^2 = \sigma_{T_{oxpL}}^2 + \sigma_{T_{oxnL}}^2 + \sigma_{\mu_{pG}}^2 + \sigma_{\mu_{pL}}^2 + \sigma_{\mu_{nG}}^2 + \sigma_{\mu_{nL}}^2 + \sigma_{\varepsilon_{\beta p}}^2 + \sigma_{\varepsilon_{\beta n}}^2$$

where the variation of the global common parameter  $T_{ox}$  is taken to be perfectly correlated between the PMOS and NMOS devices (assumed to be measured from adjacent structures), and the variations of independent global parameters and, by definition, all local parameters (and measurement errors) are taken to be independent. Note that in (4) through (6) the variations are all in relative terms, i.e. percentages, rather than absolute terms, and are normalized to the means for each quantity, and although not explicitly shown the local variations depend on geometry. Also, both global and local components of variation are included, indicated by the final  $G$  or  $L$  in the subscript, as are the measurement errors  $\varepsilon_{\beta_p}$  and  $\varepsilon_{\beta_n}$ .

The key insight in forming and analyzing the ratio  $\beta_r$  is that the global component of variation of  $T_{ox}$  is common to both  $\beta_p$  and  $\beta_n$ , and so cancels when the ratio is formed, thereby allowing information about the variation of the underlying physical process parameters to be discerned. From (4) through (6)

$$(7) \quad \sigma_{T_{oxG}}^2 = 0.5(\sigma_{\beta_p}^2 + \sigma_{\beta_n}^2 - \sigma_{\beta_r}^2)$$

$$(8) \quad \sigma_{\mu_{pG}}^2 + \sigma_{\mu_{pL}}^2 + \sigma_{T_{oxpL}}^2 + \sigma_{\varepsilon_{\beta p}}^2 = 0.5(\sigma_{\beta_p}^2 - \sigma_{\beta_n}^2 + \sigma_{\beta_r}^2)$$

$$(9) \quad \sigma_{\mu_{nG}}^2 + \sigma_{\mu_{nL}}^2 + \sigma_{T_{oxnL}}^2 + \sigma_{\varepsilon_{\beta n}}^2 = 0.5(\sigma_{\beta_n}^2 - \sigma_{\beta_p}^2 + \sigma_{\beta_r}^2)$$

and if the device geometries are large, so that the local components of variation are small, and the measurements errors are small, then (8) and (9) are simply  $\sigma_{\mu_{pG}}^2$  and  $\sigma_{\mu_{nG}}^2$ , respectively.

There are two interesting and important ramifications of the analysis above. First, it allows the variance of (the global component of)  $T_{ox}$  to be measured from simple DC measurements, and does not require any capacitance or  $S$ -parameter data. The only previously proposed DC technique we are aware of to determine  $T_{ox}$  is based on Fowler-Nordheim tunneling [3], which is more complex and involves a greater degree of model complexity than using  $\beta$ . (Body effect data of course allow  $T_{ox}$  to be characterized, however this requires knowledge of  $N$ .) Second, note that measurement errors do not affect the calculation of the variance of  $T_{ox}$ . All local components of variation and all measurement errors are

ascribed to the mobility variances. Therefore our technique gives an error-free value for the variance of  $T_{ox}$ . We are not aware of any previous technique that has this property.

Table 1 shows numerical results based on the above analysis, where the local variations and measurement errors are taken to be negligible, so the values are for the global components of variation. The values in the first column are calculated from the data of Fig. 2, the values in the second column are calculated from (7) through (9), and the values in the third column are calculated from a 10,000 sample Monte Carlo simulation based on the model (2) and (3), using the standard deviation values in the second column. From the model, the correlation is calculated as

$$(10) \quad \rho_{\beta_p\beta_n} = \frac{\sigma_{T_{ox}}^2}{\sqrt{(\sigma_{\mu_p}^2 + \sigma_{T_{ox}}^2)(\sigma_{\mu_n}^2 + \sigma_{T_{ox}}^2)}}$$

Variances are taken to include only the global component.

Measured	Modeled	MC Simulation
$\sigma_{\beta_p} = 1.315\%$	$\sigma_{\mu_p} = 0.688\%$	$\sigma_{\beta_p} = 1.289\%$
$\sigma_{\beta_n} = 1.232\%$	$\sigma_{\mu_n} = 0.512\%$	$\sigma_{\beta_n} = 1.223\%$
$\sigma_{\beta_r} = 0.857\%$	$\sigma_{T_{ox}} = 1.121\%$	$\sigma_{\beta_r} = 0.842\%$
$\rho_{\beta_p\beta_n} = 0.7748$	$\rho_{\beta_p\beta_n} = 0.7754$	$\rho_{\beta_p\beta_n} = 0.7768$

**Table 1.**  $\beta$  Analysis Results

The accuracy of reproduction of the statistics, including correlation, is clear, even though the model is based on uncorrelated parameters; the correlation follows from the functional dependence of the device parameters ( $\beta_p$ ,  $\beta_n$ , and  $\beta_r$ ) on the independent model parameters ( $\mu_p$ ,  $\mu_n$ , and  $T_{ox}$ ).

At each site measured for the DC data,  $T_{ox}$  was also calculated from C(V) data on large area structures. The  $\sigma_{T_{ox}}$  from direct measurement was 1.246% from PMOS devices and 1.219% from NMOS devices. This agrees well with value of 1.121% computed from the DC  $\beta$  data. As should be expected, the directly measured values are larger, as they include both measurement errors and local variations, which, as per (7), are not included in the  $\sigma_{T_{ox}}$  calculated from the DC  $\beta$  data.

#### 4. STATISTICAL MODELING AND CHARACTERIZATION USING DIFFERENCES

The example in the previous section was based on analyzing a standard part of the formulation of most MOSFET models, for which  $T_{ox}$ ,  $\mu_p$  and  $\mu_n$  (or equivalents) are parameters (and linking the  $T_{ox}$  parameters of PMOS and NMOS models to the same global statistical variable, while keeping the individual PMOS and NMOS mobilities as separate statistical parameters). However, the initial impetus that drove the development of the techniques we report here was the need to model correlations between PMOS and NMOS  $L_{eff}$ . These lengths follow from the two model parameters  $\Delta L_p$  and  $\Delta L_n$ , and at first we had, despite our objections outlined in the

Introduction, resigned ourselves to having to resort to defining statistical models in terms of numerical (i.e. empirical) correlations between these two parameters. However, we then realized it was still possible to model correlations in PMOS and NMOS  $L_{eff}$  physically, the trick was in how the individually unobservable components of  $L_{eff}$  for each device type could be characterized.

Physically, two major effects control  $L_{eff}$ ; the gate polysilicon critical dimension ( $C_d$ , i.e. poly etch, highly correlated between PMOS and NMOS), and the source-drain junction formations. By considering all contributions completely correlated between PMOS and NMOS  $L_{eff}$  to be embodied in a common poly  $C_d$ , and all independent contributions to be parts of overall out-diffusions  $O_{dp}$  and  $O_{dn}$  that are separate for PMOS and NMOS,

$$(11) \quad \Delta_{Lp} = C_d + O_{dp},$$

$$(12) \quad \Delta_{Ln} = C_d + O_{dn}.$$

For the purpose of characterization, besides the direct channel length offsets (11) and (12), we can form the difference between them

$$(13) \quad \Delta_{\Delta L} = \Delta_{Lp} - \Delta_{Ln}.$$

Proceeding as before, considering global and local variations in the underlying process parameters, and measurement errors  $\epsilon$ , we have

$$(14) \quad \sigma_{\Delta_{Lp}}^2 = \sigma_{C_dG}^2 + \sigma_{C_{dpL}}^2 + \sigma_{O_{dpG}}^2 + \sigma_{O_{dpL}}^2 + \sigma_{\epsilon_{\Delta_{Lp}}}^2$$

$$(15) \quad \sigma_{\Delta_{Ln}}^2 = \sigma_{C_dG}^2 + \sigma_{C_{dnL}}^2 + \sigma_{O_{dnG}}^2 + \sigma_{O_{dnL}}^2 + \sigma_{\epsilon_{\Delta_{Ln}}}^2$$

$$(16) \quad \sigma_{\Delta_{\Delta L}}^2 = \sigma_{C_{dpL}}^2 + \sigma_{C_{dnL}}^2 + \sigma_{O_{dpG}}^2 + \sigma_{O_{dpL}}^2 + \sigma_{O_{dnG}}^2 + \sigma_{O_{dnL}}^2 + \sigma_{\epsilon_{\Delta_{Lp}}}^2 + \sigma_{\epsilon_{\Delta_{Ln}}}^2.$$

Unlike for the case of the gain factor, the variances in (14) through (16) are not normalized, and are in units of square dimension and not percent. Again, forming appropriate combinations gives

$$(17) \quad \sigma_{C_dG}^2 = 0.5(\sigma_{\Delta_{Lp}}^2 + \sigma_{\Delta_{Ln}}^2 - \sigma_{\Delta_{\Delta L}}^2)$$

$$(18) \quad \sigma_{O_{dpG}}^2 + \sigma_{O_{dpL}}^2 + \sigma_{C_{dpL}}^2 + \sigma_{\epsilon_{\Delta_{Lp}}}^2 = 0.5(\sigma_{\Delta_{Lp}}^2 - \sigma_{\Delta_{Ln}}^2 + \sigma_{\Delta_{\Delta L}}^2)$$

$$(19) \quad \sigma_{O_{dnG}}^2 + \sigma_{O_{dnL}}^2 + \sigma_{C_{dnL}}^2 + \sigma_{\epsilon_{\Delta_{Ln}}}^2 = 0.5(\sigma_{\Delta_{Ln}}^2 - \sigma_{\Delta_{Lp}}^2 + \sigma_{\Delta_{\Delta L}}^2).$$

Assuming that local variations and measurement errors are small compared to global variations, then (18) and (19) give  $\sigma_{O_{dpG}}^2$  and  $\sigma_{O_{dnG}}^2$ , respectively.

Note that for one specific measurement, and not in a statistical sense, (11) through (13) give

$$(20) \quad \begin{bmatrix} \Delta_{Lp} \\ \Delta_{Ln} \\ \Delta_{\Delta L} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} C_d \\ O_{dp} \\ O_{dn} \end{bmatrix}$$

which is obviously singular and does not allow for unique solution of  $C_d$ ,  $O_{dp}$  and  $O_{dn}$ . When applied to statistical variances, the entries in the coefficient matrix of (20) are squared, and it becomes well conditioned.

Applying the above analysis to measured  $L_{eff}$  data gave the results of Table 2. The values in the first column are calculated from the data of Fig. 1, the values in the second column are calculated from (17) through (19), assuming the measurement errors and local components of variation are negligible, and the values in the third column are calculated from a 10,000 sample Monte Carlo simulation based on the model (11) through (13), using the standard deviation values in the third column. From the model, the correlation is calculated as

$$(21) \quad \rho_{LpLn} = \frac{\sigma_{C_d}^2}{\sqrt{(\sigma_{\Delta_{Lp}}^2 + \sigma_{C_d}^2)(\sigma_{\Delta_{Ln}}^2 + \sigma_{C_d}^2)}}.$$

Variances are taken to include only the global component.

Measured	Modeled	MC Simulation
$\sigma_{\Delta_{Lp}} = 0.02999$	$\sigma_{O_{dp}} = 0.00771$	$\sigma_{\Delta_{Lp}} = 0.02973$
$\sigma_{\Delta_{Ln}} = 0.03518$	$\sigma_{O_{dn}} = 0.01993$	$\sigma_{\Delta_{Ln}} = 0.03473$
$\sigma_{\Delta_{\Delta L}} = 0.02137$	$\sigma_{C_d} = 0.02899$	$\sigma_{\Delta_{\Delta L}} = 0.02106$
$\rho_{LpLn} = 0.7964$	$\rho_{LpLn} = 0.7964$	$\rho_{LpLn} = 0.7972$

**Table 2.**  $L_{eff}$  Analysis Results ( $\sigma$ 's are in units of  $\mu\text{m}$ )

The 1-, 2-, and 3-sigma contours (ellipses) from the Monte Carlo samples are included in Fig. 1. From both the values in Table 2 and the contours of Fig. 1, it is apparent that the correlation structure of the data is modeled well, using statistically independent parameters.

Unlike the case for  $\beta$ , modeling of the correlation of PMOS and NMOS  $L_{eff}$  required introduction of additional model parameters. These are implemented in practice using sub-circuits. (We applied a similar analysis to PMOS and NMOS  $W_{eff}$ , but found them to be so highly correlated that we did not need to introduce separate parameters and could just use a common  $\Delta_W$  for both PMOS and NMOS models.)

## 5. BODY EFFECT ANALYSIS

From DC data alone it is not feasible to separate the effects of  $T_{ox}$  and  $N$ . For the gain factor this is because  $T_{ox}$  is confounded with mobility, for threshold voltage this is because both parameters, as well as flatband voltage, affect  $V_{t0}$ , and for the body effect this is because both factors are confounded in the body effect coefficient  $\gamma = \sqrt{2q\epsilon_{Si}N} T_{ox} / \epsilon_{ox}$ . However, noting from this that  $T_{ox}$  is correlated between PMOS and

NMOS, but  $N_p$  and  $N_n$  are independent, then forming the ratio of body effect coefficients of PMOS and NMOS devices should allow separation of the underlying parameters.

In our fab measurement data there is also the difference between threshold voltage at high and zero substrate bias,

$$(22) \quad D = V_{th}(V_{sb}) - V_{th}(0) = \gamma \left( \sqrt{2\phi_f + V_{sb}} - \sqrt{2\phi_f} \right)$$

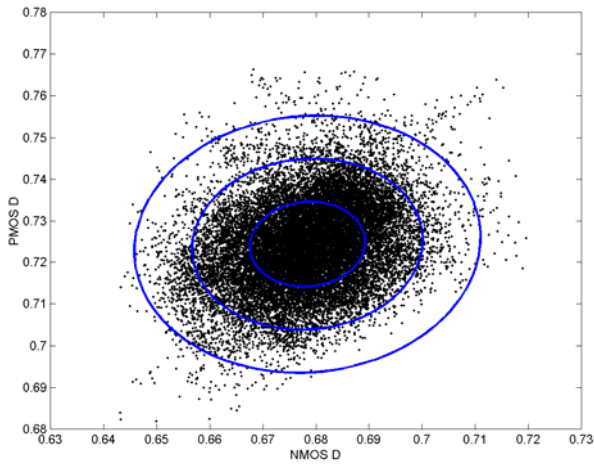
where  $\phi_f = kT \ln(N/n_i)/q$ . The ratio of these differences is  $D_r = D_p/D_n$ . Assuming the local components of variation and the measurement errors are small gives, for the global components of variation, in normalized terms,

$$(23) \quad \begin{bmatrix} \sigma_{D_p}^2 \\ \sigma_{D_n}^2 \\ \sigma_{D_r}^2 \end{bmatrix} = \begin{bmatrix} \left( \frac{N_p}{D_p} \frac{\partial D_p}{\partial N_p} \right)^2 & 0 & 1 \\ 0 & \left( \frac{N_n}{D_n} \frac{\partial D_n}{\partial N_n} \right)^2 & 1 \\ \left( \frac{N_p}{D_r} \frac{\partial D_r}{\partial N_p} \right)^2 & \left( \frac{N_n}{D_r} \frac{\partial D_r}{\partial N_n} \right)^2 & 0 \end{bmatrix} \begin{bmatrix} \sigma_{N_p}^2 \\ \sigma_{N_n}^2 \\ \sigma_{T_{ox}}^2 \end{bmatrix}.$$

The (normalized) sensitivities in (23) need to be evaluated from a model. For this,  $T_{ox}$  is set to the mean measured value, and the nominal values of  $N_p$  and  $N_n$  are calculated (using  $T_{ox}$ ) from the mean measure values of  $D_p$  and  $D_n$ , respectively. The sensitivities are then calculated from (22).

Measured	Modeled	MC Simulation
$\sigma_{D_p} = 1.481\%$	$\sigma_{N_p} = 2.515\%$	$\sigma_{D_p} = 1.457\%$
$\sigma_{D_n} = 1.543$	$\sigma_{N_n} = 2.669\%$	$\sigma_{D_n} = 1.532$
$\sigma_{D_r} = 1.782\%$	$\sigma_{T_{ox}} = 0.8364\%$	$\sigma_{D_r} = 1.766\%$
$\rho_{D_p D_n} = 0.3078$	$\rho_{D_p D_n} = 0.3061$	$\rho_{D_p D_n} = 0.3039$

**Table 3.** Body Effect Analysis Results



**Fig. 3** Correlation between PMOS  $D$  and NMOS  $D$ . Points are measured data, ellipses are from fitted model.

Table 3 shows numerical results from analysis of the measured body effect data, and Fig. 3 shows the measured data and 1-, 2-, and 3-sigma contours from the Monte Carlo samples. The  $\sigma_{T_{ox}}$  is lower than that calculated from the  $\beta$  analysis; the model used here is less accurate (threshold is an imprecisely defined quantity, constant doping is assumed) and the correlation smaller hence the reliability of extraction of the variance of the correlated parameter  $T_{ox}$  is less. Nevertheless it does show that, to some degree of accuracy, the effects of  $T_{ox}$  and  $N$  can be determined statistically from DC data.

## 6. CONCLUSIONS

We have shown that correlations between electrical performances of different devices that have some partial underlying physical commonality can be modeled by identifying or introducing appropriate independent and common underlying physical parameters, and the statistics of these parameters can be determined from appropriate combinations (ratios or differences) of measurements that are sensitive to variations in the underlying physical parameters.

At first it seems counter intuitive that 3 parameters can be determined from 2 measurements. For a single instance they cannot. But statistically the information content in the derived measurement, e.g.  $\beta_r = \beta_p/\beta_n$  or  $\Delta_{\Delta L} = \Delta_{Lp} - \Delta_{Ln}$ , is equivalent to the correlation of the original measurements. That is what allows the 3<sup>rd</sup> parameter to be determined.

We have analyzed models and data for MOSFET parameters, however the technique is general and can be applied to any quantities that are partially correlated between different devices whose behavior depends on some common and some separate underlying global process parameters.

Although a common framework of variance analysis has been shown to be applicable to both global and local variation characterization [4], this is not the case for correlation modeling. By definition, the local component of variation is uncorrelated between devices. Therefore the procedure described here is only applicable for analysis of the global component of variation, and should only be applied to data for which this dominates the local component of variation.

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